

# Pseudoscalar Meson and Charmed Baryon Scattering Lengths

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We have calculated the scattering lengths between the pseudoscalar meson and charmed triplet, sextet, and excited sextet baryon to the third order with the heavy baryon chiral perturbation theory. The chiral expansion of some pion and eta channels converges well.

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## I. INTRODUCTION

Up to now many heavy hadrons have been discovered experimentally. Their inner structures and interactions attract much attention. Some of them are speculated to be possible new hadron states beyond the traditional quark model. For example, the newly discovered  $Z_b$  states are treated as the  $\bar{B}^{(*)}B^*$  molecular states [1, 2]. Many other molecular states, such as those composed of  $\Xi_c\Xi_c$ ,  $D\bar{D}^*$ , are also proposed [3–8]. Whether there is attractive interaction between the particles is the most important condition to form molecular states.

On the other hand, the hadron-hadron interaction may distort the conventional quark model spectrum through the coupled channel effects. For example, the bare charm-strange scalar meson lies around 2.4 GeV according to the quark model calculation. Experimentally the mass of the  $D_{s0}(2317)$  state was measured to be around 2.3 GeV. The attractive interaction between the  $D$  meson and kaon is essential to lower its mass through the coupling effect between the bare  $c\bar{s}$  state and the  $DK$  continuum [9].

There has been lots of research work on the strong interactions of the charmed or bottomed mesons, such as the lattice study, calculations with the chiral perturbation theory and so on [10–19]. The charmed triplet ( $B_3$ ), sextet ( $B_6$ ) and excited sextet ( $B_6^*$ ) baryons are relatively stable particles. The pair from the ground and corresponding excited sextet form a degenerate doublet in the heavy quark spin symmetry limit. They interact with other particles through the exchange of the pseudoscalar mesons in the low energy effective field theory. It is very important to study the strong interaction between the lightest pseudoscalar meson ( $\phi$ ) and charmed baryon.

A physical observable such as the scattering amplitude can be expanded order by order with the explicit power counting in the heavy baryon chiral perturbation theory (HB $\chi$ PT). The inclusion of the nonanalytic corrections resulting from the loop diagrams would highly reduce the error of extraction in the lattice study [20, 21], which is one of the motivations of the present investigation.

In this work, we shall study the pseudoscalar meson and charmed baryon scattering lengths to the third order with HB $\chi$ PT. We include the interaction of  $B_6$  and  $B_6^*$  explicitly for the  $\phi B_3$  scattering instead of absorbing their effects into the low energy constants (LECs) at higher order since the mass difference among the charmed baryons is small and the couplings between them can not be neglected. The situation is similar for the  $\phi B_6$  and  $\phi B_6^*$  cases. We express the results as power series in  $\epsilon = p/\Lambda_\chi$  with the explicit power counting, where  $p$  represents the mass and momentum of pseudoscalar Goldstone bosons, the residual momentum of charmed baryons, the mass difference among charmed baryons, while  $\Lambda_\chi$  is either the mass of charmed baryons or  $4\pi f$ .

This paper is organized as follows: in Sec. II, we list the HB $\chi$ PT Lagrangians of the pseudoscalar mesons and charmed baryons, with which we get the expressions of the  $T$ -matrices at thresholds. In Sec. III, we estimate the LECs in the Lagrangians. We present the numerical results and discussions in Sec. IV. Sec. V is a short conclusion.

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## II. THE $T$ -MATRICES AT THRESHOLDS

The average mass of the charmed triplet baryons  $M_0(2408 \text{ MeV})$ , which provides the base when we refer to the mass difference in the following. The HB $\chi$ PT Lagrangians at the leading order read

$$\mathcal{L}_{\phi\phi}^{(2)} = f^2 \text{Tr} \left( u_\mu u^\mu + \frac{\chi_+}{4} \right), \quad (1)$$

$$\begin{aligned} \mathcal{L}_{B\phi}^{(1)} = & \frac{1}{2} \text{Tr}[\bar{B}_3 i v \cdot D B_3] + \text{Tr}[\bar{B}_6 (i v \cdot D - \delta_2) B_6] - \text{Tr}[\bar{B}_6^* (i v \cdot D - \delta_3) B_6^*] \\ & + 2g_1 \text{Tr}(\bar{B}_6 S \cdot u B_6) + 2g_2 \text{Tr}(\bar{B}_6 S \cdot u B_3 + \text{H.c.}) + g_3 \text{Tr}(\bar{B}_{6\mu}^* u^\mu B_6 + \text{H.c.}) \\ & + g_4 \text{Tr}(\bar{B}_{6\mu}^* u^\mu B_3 + \text{H.c.}) + 2g_5 \text{Tr}(\bar{B}_6^* S \cdot u B_6^*) + 2g_6 \text{Tr}(\bar{B}_3 S \cdot u B_3), \end{aligned} \quad (2)$$

where  $v_\mu$  is the velocity of a slowly moving baryon,  $S_\mu$  is the spin matrix,  $g_i$  is the coupling of the  $BB\phi$ -vertex, and  $\delta_i$  is the mass difference between the charmed baryons,

$$\delta_1 = M_{B_6^*} - M_{B_6} = 67 \text{ MeV}, \quad \delta_2 = M_{B_6} - M_{B_3} = 127 \text{ MeV}, \quad \delta_3 = M_{B_6^*} - M_{B_3} = 194 \text{ MeV}. \quad (3)$$

The field notations are

$$\phi = \sqrt{2} \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}, \quad B_3 = \begin{pmatrix} 0 & \Lambda_c^+ & \Xi_c^+ \\ -\Lambda_c^+ & 0 & \Xi_c^0 \\ -\Xi_c^+ & -\Xi_c^0 & 0 \end{pmatrix}, \quad B_6 = \begin{pmatrix} \Sigma_c^{++} & \frac{\Sigma_c^+}{\sqrt{2}} & \frac{\Xi_c'^+}{\sqrt{2}} \\ \frac{\Sigma_c^+}{\sqrt{2}} & \Sigma_c^0 & \frac{\Xi_c'^0}{\sqrt{2}} \\ \frac{\Xi_c'^+}{\sqrt{2}} & \frac{\Xi_c'^0}{\sqrt{2}} & \Omega_c^0 \end{pmatrix}, \quad (4)$$

$$\Gamma_\mu = \frac{i}{2} [\xi^\dagger, \partial_\mu \xi], \quad u_\mu = \frac{i}{2} \{\xi^\dagger, \partial_\mu \xi\}, \quad \xi = \exp(i \frac{\phi}{f}), \quad \chi_\pm = \xi^\dagger \chi \xi^\dagger \pm \xi \chi \xi, \quad \chi = \text{diag}(m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2), \quad (5)$$

and the definition of  $B_6^*$  is similar to that of  $B_6$ . The covariant derivatives,  $iD^\mu B_{ab} = i\partial^\mu B_{ab} + \Gamma_a^d B_{db} + \Gamma_b^d B_{ad}$ , will generate the  $BB\phi$ -vertexes.

The Lagrangian at  $O(\epsilon^2)$  contains the counter terms and the recoil terms. The counter terms are constructed on the basis of the chiral and other symmetries and proportional to  $\bar{c}_i, c_i, \tilde{c}_i$  in Eq. (6). The recoil terms are derived from the Lagrangians of the leading order and proportional to  $g_i^2$ . We list the relevant terms below,

$$\begin{aligned} \mathcal{L}_{B\phi}^{(2)} = & \bar{c}_0 \text{Tr}[\bar{B}_3 B_3] \text{Tr}[\chi_+] + \bar{c}_1 \text{Tr}[\bar{B}_3 \tilde{\chi}_+ B_3] + \left( \bar{c}_2 - \frac{2g_6^2 + g_2^2}{4M_0} \right) \text{Tr}[\bar{B}_3 v \cdot u v \cdot u B_3] \\ & + \left( \bar{c}_3 - \frac{2g_6^2 - g_2^2}{4M_0} \right) \bar{B}_3^{ab} v \cdot u_a^c v \cdot u_b^d B_{3,cd} \\ & + c_0 \text{Tr}[\bar{B}_6 B_6] \text{Tr}[\chi_+] + c_1 \text{Tr}[\bar{B}_6 \tilde{\chi}_+ B_6] + \left( c_2 - \frac{2g_2^2 + g_1^2}{4M_0} \right) \text{Tr}[\bar{B}_6 v \cdot u v \cdot u B_6] \\ & + \left( c_3 + \frac{2g_2^2 - g_1^2}{4M_0} \right) \bar{B}_6^{ab} v \cdot u_a^c v \cdot u_b^d B_{6,cd} + c_4 \text{Tr}[\bar{B}_6 B_6] \text{Tr}[v \cdot u v \cdot u] \\ & - \tilde{c}_0 \text{Tr}[\bar{B}_6^* B_6^*] \text{Tr}[\chi_+] - \tilde{c}_1 \text{Tr}[\bar{B}_6^* \tilde{\chi}_+ B_6^*] - \left( \tilde{c}_2 - \frac{g_5^2}{4M_0} \right) \text{Tr}[\bar{B}_6^* v \cdot u v \cdot u B_6^*] \\ & - \left( \tilde{c}_3 - \frac{g_5^2}{4M_0} \right) \bar{B}_6^{*ab} v \cdot u_a^c v \cdot u_b^d B_{6,cd}^* - \tilde{c}_4 \text{Tr}[\bar{B}_6^* B_6^*] \text{Tr}[v \cdot u v \cdot u], \end{aligned} \quad (6)$$

where the traceless  $\tilde{\chi}_\pm$  are defined as:  $\tilde{\chi}_\pm = \chi_\pm - \frac{1}{3} \text{Tr}[\chi_\pm]$ .

The Lagrangian at  $O(\epsilon^3)$  also contains the recoil-term part,

$$\begin{aligned} \mathcal{L}_{B\phi}^{(3,r)} = & \frac{2g_6^2 + g_2^2}{8M_0^2} \text{Tr}[\bar{B}_3 v \cdot u (i v \cdot D) v \cdot u B_3] + \frac{2g_6^2 - g_2^2}{8M_0^2} \bar{B}_3^{ab} v \cdot u_a^c (i v \cdot D) v \cdot u_b^d B_{3,cd} + \frac{g_2^2}{8M_0^2} \text{Tr}[\bar{B}_3 v \cdot u v \cdot u B_3] \delta_2 \\ & - \frac{g_2^2}{8M_0^2} \bar{B}_3^{ab} v \cdot u_a^c v \cdot u_b^d B_{3,cd} \delta_2 + \frac{2g_2^2 + g_1^2}{8M_0^2} \text{Tr}[\bar{B}_6 v \cdot u (i v \cdot D) v \cdot u B_6] \\ & - \frac{2g_2^2 - g_1^2}{8M_0^2} \bar{B}_6^{ab} v \cdot u_a^c (i v \cdot D) v \cdot u_b^d B_{6,cd} + \frac{g_1^2}{8M_0^2} \text{Tr}[\bar{B}_6 v \cdot u v \cdot u B_6] \delta_2 + \frac{g_1^2}{8M_0^2} \bar{B}_6^{ab} v \cdot u_a^c v \cdot u_b^d B_{6,cd} \delta_2 \end{aligned}$$

$$-\frac{g_5^2}{8M_0^2}\text{Tr}[\bar{B}_6^*v \cdot u(iv \cdot D + \delta_3)v \cdot u B_6^*] - \frac{g_5^2}{8M_0^2}\bar{B}_6^{*ab}v \cdot u_a^c(iv \cdot D + \delta_3)v \cdot u_b^d B_{6,cd}^* \quad (7)$$

We neglect the contributions from the finite counter-term part at  $O(\epsilon^3)$  as in Refs. [22, 23], and cancel the divergences of the loop diagrams with the following infinite part

$$\begin{aligned} \mathcal{L}_{B\phi}^{(3,c)} = & \frac{3}{4f^2}L\text{Tr}[\bar{B}_3[v \cdot u, \tilde{\chi}_-]B_3] - (\frac{5\alpha g_2^2\delta_2}{2f^2} - \frac{5\beta g_4^2\delta_3}{8f^2})L\text{Tr}[\bar{B}_3\tilde{\chi}_+B_3] + (\frac{10\alpha g_2^2\delta_2}{3f^2} - \frac{5\beta g_4^2\delta_3}{6f^2})L\text{Tr}[\bar{B}_3v \cdot u v \cdot u B_3] \\ & - (\frac{32\alpha g_2^2\delta_2}{3f^2} - \frac{8\beta g_4^2\delta_3}{3f^2})L\bar{B}_3^{ab}v \cdot u_a^c v \cdot u_b^d B_{3,cd} \\ & + \frac{3}{2f^2}L\text{Tr}[\bar{B}_6[v \cdot u, \tilde{\chi}_-]B_6] + (\frac{\alpha g_2^2\delta_2}{f^2} + \frac{7\beta g_3^2\delta_1}{24f^2})L\text{Tr}[\bar{B}_6\tilde{\chi}_+B_6] + (\frac{4\alpha g_2^2\delta_2}{f^2} + \frac{5\beta g_3^2\delta_1}{2f^2})L\text{Tr}[\bar{B}_6v \cdot u v \cdot u B_6] \\ & + (\frac{8\alpha g_2^2\delta_2}{f^2} - \frac{\beta g_3^2\delta_1}{f^2})L\bar{B}_6^{ab}v \cdot u_a^c v \cdot u_b^d B_{6,cd} - (\frac{4\alpha g_2^2\delta_2}{3f^2} + \frac{11\beta g_3^2\delta_1}{18f^2})L\text{Tr}[\bar{B}_6B_6]\text{Tr}[v \cdot u v \cdot u] \\ & - \frac{3}{2f^2}L\text{Tr}[\bar{B}_6^*[v \cdot u, \tilde{\chi}_-]B_6^*] + (\frac{7g_3^2\delta_1}{24f^2} + \frac{g_4^2\delta_3}{4f^2})L\text{Tr}[\bar{B}_6^*\tilde{\chi}_+B_6^*] + (\frac{5g_3^2\delta_1}{2f^2} + \frac{g_4^2\delta_3}{f^2})L\text{Tr}[\bar{B}_6^*v \cdot u v \cdot u B_6^*] \\ & - (\frac{g_3^2\delta_1}{f^2} - \frac{2g_4^2\delta_3}{f^2})L\bar{B}_6^{*ab}v \cdot u_a^c v \cdot u_b^d B_{6,cd}^* - (\frac{11g_3^2\delta_1}{18f^2} + \frac{g_4^2\delta_3}{3f^2})L\text{Tr}[\bar{B}_6^*B_6^*]\text{Tr}[v \cdot u v \cdot u], \end{aligned} \quad (8)$$

where in the  $D$  dimensional space-time,

$$\alpha = S^2 = -3/4 - (D-4)/4, \quad \beta = P^{3/2\mu}_\mu = 2 + (D-4), \quad L = \frac{\lambda^{D-4}}{16\pi^2} \left\{ \frac{1}{D-4} + \frac{1}{2}(\gamma_E - 1 - \ln 4\pi) \right\}, \quad (9)$$

$P_{\mu\nu}^{3/2}$  is the projection operator for the Rarita-Schwinger field,  $\gamma_E = 0.5772157$  is the Euler constant,  $\lambda = 4\pi f$  is the energy scale.

The scattering length  $a_{\phi B}$  is related to the threshold  $T$ -matrix  $T_{\phi B}$  by  $T_{\phi B} = 4\pi(1 + m_\phi/m_B)a_{\phi B}$ . At the leading order, only the  $BB\phi\phi$ -vertex from the contact terms in  $\mathcal{L}_{B\phi}^{(1)}$  contributes to the  $T$ -matrices at the threshold. At the second order, the corresponding  $BB\phi\phi$ -vertex of  $\mathcal{L}_{B\phi}^{(2)}$  contributes. At the third order, in addition to the contribution of  $\mathcal{L}_{B\phi}^{(3,r/c)}$ , the  $T$ -matrices also receive the contribution from the loop diagrams consisting of the vertices in the leading order Lagrangian.

The nonvanishing loop diagrams are shown in Fig. 1. Since there is no vertex like  $B_3B_6\phi\phi$  at the leading order, the charmed baryons in different representations do not appear in the diagrams (I) as intermediate states. But they appear in the diagrams (II) through the axial coupling.

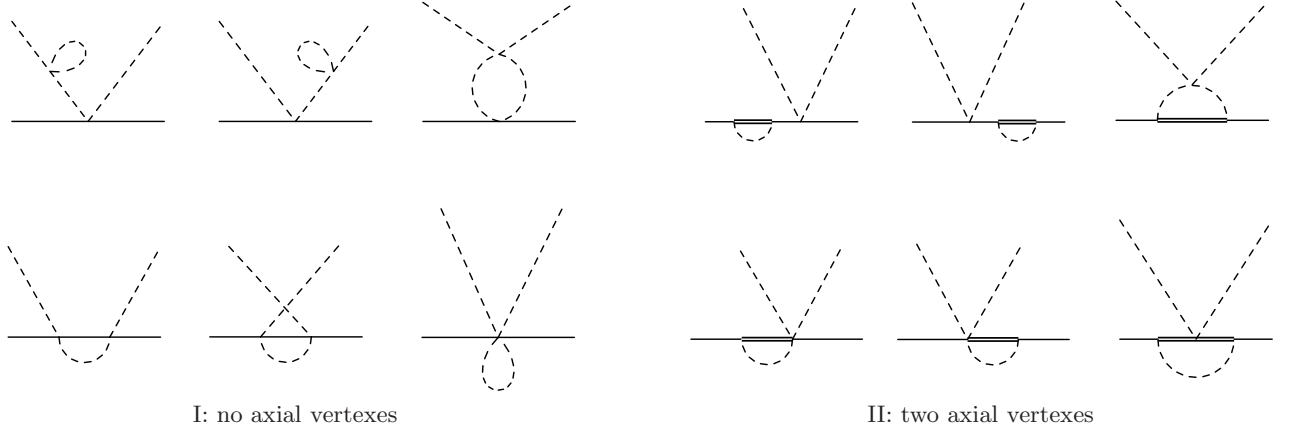


FIG. 1: Nonvanishing loop diagrams for the pseudoscalar meson and charmed meson scattering lengths to  $O(\epsilon^3)$  with HB $\chi$ PT. The dashed lines represent the pseudoscalar Goldstone bosons. Both the thin solid lines and thick solid lines represent charmed baryons. The internal thin solid lines represent the charmed baryons in the same representation as the external baryons while the internal thick solid lines represent all possible charmed baryons.

We calculate the loop diagrams with dimensional regularization and the modified minimal subtraction scheme. We use the LECs in Eq. (8) to cancel the divergence. At last we express the  $T$ -matrices in terms of  $f_\phi$  rather than  $f$  with the help of their relation in Refs. [24, 25].

The isospin symmetry is explicitly kept throughout our calculation. So we only list the 49 isospin-independent  $T$ -matrices in the following subsections.

### A. $\phi B_{\bar{3}}$ scattering

We list the  $T$ -matrices for the pseudoscalar meson and  $\bar{3}$  charmed baryon scattering, separate them order by order with braces, and distinguish between the tree and loop diagram contribution at  $O(\epsilon^3)$  with square brackets,

$$\begin{aligned}
T_{\pi\Lambda_c}^{(1)} &= \{0\} + \left\{ -\frac{m_\pi^2(24\bar{c}_0 + 4\bar{c}_1 - 3\bar{C}_2 - 3\bar{C}_3)}{3f_\pi^2} \right\} + \left\{ \left[ \frac{m_\pi^2\delta_2(\bar{d}_2 + \bar{d}_3)}{f_\pi^2 M_0^2} \right] + \left[ -\frac{1}{16}V(m_K^2, -m_\pi) - \frac{1}{16}V(m_K^2, m_\pi) \right. \right. \\
&\quad \left. \left. - \frac{4}{9}m_\pi^2 Y_1(m_\eta) + 2m_\pi^2 W_1(m_\pi) \right] \right\}, \\
T_{\pi\Xi_c}^{(1/2)} &= \left\{ \frac{m_\pi}{f_\pi^2} \right\} + \left\{ -\frac{m_\pi^2(48\bar{c}_0 - 4\bar{c}_1 - 3\bar{C}_2)}{6f_\pi^2} \right\} + \left\{ \left[ \frac{m_\pi^2(16\bar{d}_1 m_\pi + \bar{d}_2 \delta_2)}{2f_\pi^2 M_0^2} \right] + \left[ \frac{1}{32}V(m_K^2, -m_\pi) - \frac{3}{32}V(m_K^2, m_\pi) \right. \right. \\
&\quad \left. \left. - \frac{1}{4}V(m_\pi^2, m_\pi) - \frac{1}{2}m_\pi^2 W_1(m_\eta) - \frac{1}{9}m_\pi^2 Y_1(m_\eta) + \frac{1}{2}m_\pi^2 W_1(m_\pi) + m_\pi^2 Y_1(m_\pi) \right] \right\}, \\
T_{\pi\Xi_c}^{(3/2)} &= \left\{ -\frac{m_\pi}{2f_\pi^2} \right\} + \left\{ -\frac{m_\pi^2(48\bar{c}_0 - 4\bar{c}_1 - 3\bar{C}_2)}{6f_\pi^2} \right\} + \left\{ \left[ -\frac{m_\pi^2(8\bar{d}_1 m_\pi - \bar{d}_2 \delta_2)}{2f_\pi^2 M_0^2} \right] + \left[ -\frac{1}{16}V(m_K^2, -m_\pi) \right. \right. \\
&\quad \left. \left. - \frac{3}{16}V(m_\pi^2, -m_\pi) - \frac{1}{16}V(m_\pi^2, m_\pi) - \frac{1}{2}m_\pi^2 W_1(m_\eta) - \frac{1}{9}m_\pi^2 Y_1(m_\eta) + \frac{1}{2}m_\pi^2 W_1(m_\pi) + m_\pi^2 Y_1(m_\pi) \right] \right\}, \\
T_{K\Lambda_c}^{(1/2)} &= \left\{ -\frac{m_K}{2f_K^2} \right\} + \left\{ -\frac{m_K^2(48\bar{c}_0 - 4\bar{c}_1 - 3\bar{C}_2)}{6f_K^2} \right\} + \left\{ \left[ -\frac{m_K^2(8\bar{d}_1 m_K - \bar{d}_2 \delta_2)}{2f_K^2 M_0^2} \right] + \left[ -\frac{1}{16}V(m_K^2, -m_K) \right. \right. \\
&\quad \left. \left. - \frac{1}{16}V(m_K^2, m_K) - \frac{3}{32}V(m_\eta^2, -m_K) - \frac{3}{32}V(m_\pi^2, -m_K) + \frac{8}{9}m_K^2 Y_1(m_\eta) \right] \right\}, \\
T_{K\Xi_c}^{(0)} &= \left\{ \frac{m_K}{f_K^2} \right\} + \left\{ -\frac{m_K^2(24\bar{c}_0 - 8\bar{c}_1 + 3\bar{C}_3)}{3f_K^2} \right\} + \left\{ \left[ \frac{m_K^2(8\bar{d}_1 m_K - \bar{d}_3 \delta_2)}{f_K^2 M_0^2} \right] + \left[ -\frac{1}{16}V(m_K^2, -m_K) - \frac{1}{4}V(m_K^2, m_K) \right. \right. \\
&\quad \left. \left. - \frac{3}{16}V(m_\eta^2, m_K) + m_K^2 W_1(m_\eta) + \frac{2}{9}m_K^2 Y_1(m_\eta) + 2P_1 - 3U_1 \right] \right\}, \\
T_{K\Xi_c}^{(1)} &= \{0\} + \left\{ -\frac{m_K^2(24\bar{c}_0 + 4\bar{c}_1 - 3\bar{C}_2 - 3\bar{C}_3)}{3f_K^2} \right\} + \left\{ \left[ \frac{m_K^2\delta_2(\bar{d}_2 + \bar{d}_3)}{f_K^2 M_0^2} \right] + \left[ -\frac{1}{16}V(m_K^2, -m_K) - \frac{1}{16}V(m_\pi^2, m_K) \right. \right. \\
&\quad \left. \left. + m_K^2 W_1(m_\eta) + \frac{2}{9}m_K^2 Y_1(m_\eta) - \frac{2P_1}{3} + U_1 \right] \right\}, \\
T_{\eta\Lambda_c}^{(0)} &= \{0\} + \left\{ -\frac{24\bar{c}_0 m_\eta^2 - 8\bar{c}_1 m_\eta^2 + 4\bar{c}_1 m_\pi^2 - \bar{C}_2 m_\eta^2 + \bar{C}_3 m_\eta^2}{3f_\eta^2} \right\} + \left\{ \left[ \frac{m_\eta^2\delta_2(\bar{d}_2 - \bar{d}_3)}{3f_\eta^2 M_0^2} \right] + \left[ -\frac{3}{16}V(m_K^2, -m_\eta) \right. \right. \\
&\quad \left. \left. - \frac{3}{16}V(m_K^2, m_\eta) - \frac{64}{27}m_K^2 Y_1(m_\eta) + \frac{4}{3}m_K^2 W_1(m_K) + \frac{8}{3}m_K^2 Y_1(m_K) + \frac{28}{27}m_\pi^2 Y_1(m_\eta) - 2m_\pi^2 W_1(m_\pi) \right] \right\}, \\
T_{\eta\Xi_c}^{(1/2)} &= \{0\} + \left\{ -\frac{48\bar{c}_0 m_\eta^2 + 8\bar{c}_1 m_\eta^2 - 4\bar{c}_1 m_\pi^2 - 5\bar{C}_2 m_\eta^2 - 4\bar{C}_3 m_\eta^2}{6f_\eta^2} \right\} + \left\{ \left[ \frac{m_\eta^2\delta_2(5\bar{d}_2 + 4\bar{d}_3)}{6f_\eta^2 M_0^2} \right] + \left[ -\frac{3}{32}V(m_K^2, -m_\eta) \right. \right. \\
&\quad \left. \left. - \frac{3}{32}V(m_K^2, m_\eta) - \frac{8}{3}m_K^2 W_1(m_\eta) - \frac{16}{27}m_K^2 Y_1(m_\eta) + \frac{10}{3}m_K^2 W_1(m_K) + \frac{4}{3}m_K^2 Y_1(m_K) + \frac{7}{6}m_\pi^2 W_1(m_\eta) \right. \right. \\
&\quad \left. \left. + \frac{7}{27}m_\pi^2 Y_1(m_\eta) - \frac{1}{2}m_\pi^2 W_1(m_\pi) - m_\pi^2 Y_1(m_\pi) \right] \right\} \tag{10}
\end{aligned}$$

where the superscript  $I$  in  $T_{\phi B}^I$  refers to the isospin of the channel, the functions  $P$ ,  $U$ ,  $V$ ,  $W$ , and  $Y$  are listed in Appendix A, and some combination coefficients are defined as

$$\bar{C}_2 = \bar{c}_2 - \frac{2g_6^2 + g_2^2}{4M_0}, \quad \bar{C}_3 = \bar{c}_3 - \frac{2g_6^2 - g_2^2}{4M_0}, \quad \bar{d}_1 = \frac{2g_6^2 + g_2^2}{64}, \quad \bar{d}_2 = \frac{g_2^2}{8}, \quad \bar{d}_3 = -\frac{g_2^2}{8}, \tag{11}$$

We have used the Gell-Mann-Okubo mass relation  $m_\eta^2 = (4m_K^2 - m_\pi^2)/3$  to make the expression more concise.

Besides the eight  $T$ -matrices listed above, the other three isospin-independent ones can be written in terms of those in Eq. (10) by crossing symmetry,

$$T_{K\Lambda_c}^{(1/2)} = [T_{K\Lambda_c}^{(1/2)}]_{m_K \rightarrow -m_K}, \quad T_{K\Xi_c}^{(1)} = \frac{1}{2} [T_{K\Xi_c}^{(1)} + T_{K\Xi_c}^{(0)}]_{m_K \rightarrow -m_K}, \quad T_{K\Xi_c}^{(0)} = \frac{1}{2} [3T_{K\Xi_c}^{(1)} - T_{K\Xi_c}^{(0)}]_{m_K \rightarrow -m_K}. \quad (12)$$

### B. $\phi B_6$ scattering

There are 19 isospin independent  $T$ -matrices for the pseudoscalar meson and charmed sextet baryon scattering.

$$\begin{aligned} T_{\pi\Omega_c}^{(1)} &= \{0\} + \left\{ -\frac{m_\pi^2(12c_0 - 4c_1 - 3c_4)}{3f_\pi^2} \right\} + \left\{ [0] + [-\frac{1}{8}V(m_K^2, -m_\pi) - \frac{1}{8}V(m_K^2, m_\pi) - \frac{4}{9}m_\pi^2 W_2(m_\eta)] \right\}, \\ T_{\pi\Xi_c}^{(1/2)} &= \left\{ \frac{m_\pi}{f_\pi^2} \right\} + \left\{ -\frac{m_\pi^2(48c_0 - 4c_1 - 3C_2 - 12c_4)}{12f_\pi^2} \right\} + \left\{ \left[ \frac{m_\pi^2(16d_1 m_\pi + d_2 \delta_2)}{4f_\pi^2 M_0^2} \right] + [-\frac{3}{32}V(m_K^2, -m_\pi) \right. \right. \\ &\quad \left. \left. - \frac{7}{32}V(m_K^2, m_\pi) - \frac{1}{4}V(m_\pi^2, m_\pi) - \frac{1}{36}m_\pi^2 W_2(m_\eta) - \frac{1}{2}m_\pi^2 Y_2(m_\eta) + \frac{1}{4}m_\pi^2 W_2(m_\pi) + \frac{1}{2}m_\pi^2 Y_2(m_\pi)] \right\}, \\ T_{\pi\Xi_c}^{(3/2)} &= \left\{ -\frac{m_\pi}{2f_\pi^2} \right\} + \left\{ -\frac{m_\pi^2(48c_0 - 4c_1 - 3C_2 - 12c_4)}{12f_\pi^2} \right\} + \left\{ \left[ -\frac{m_\pi^2(8d_1 m_\pi - d_2 \delta_2)}{4f_\pi^2 M_0^2} \right] + [-\frac{3}{16}V(m_K^2, -m_\pi) \right. \right. \\ &\quad \left. \left. - \frac{1}{8}V(m_K^2, m_\pi) - \frac{3}{16}V(m_\pi^2, -m_\pi) - \frac{1}{16}V(m_\pi^2, m_\pi) - \frac{1}{36}m_\pi^2 W_2(m_\eta) - \frac{1}{2}m_\pi^2 Y_2(m_\eta) + \frac{1}{4}m_\pi^2 W_2(m_\pi) \right. \right. \\ &\quad \left. \left. + \frac{1}{2}m_\pi^2 Y_2(m_\pi)] \right\}, \\ T_{\pi\Sigma_c}^{(0)} &= \left\{ \frac{2m_\pi}{f_\pi^2} \right\} + \left\{ -\frac{m_\pi^2(24c_0 + 4c_1 - 3C_2 + 9C_3 - 6c_4)}{6f_\pi^2} \right\} + \left\{ \left[ \frac{m_\pi^2(16d_1 m_\pi + d_2 \delta_2 - 3d_3 \delta_2)}{2f_\pi^2 M_0^2} \right] \right. \\ &\quad \left. + [\frac{1}{16}V(m_K^2, -m_\pi) - \frac{3}{16}V(m_K^2, m_\pi) - \frac{1}{2}V(m_\pi^2, -m_\pi) - V(m_\pi^2, m_\pi) - \frac{1}{9}m_\pi^2 W_2(m_\eta) + 4m_\pi^2 W_2(m_\pi) \right. \right. \\ &\quad \left. \left. - 6m_\pi^2 Y_2(m_\pi)] \right\}, \\ T_{\pi\Sigma_c}^{(1)} &= \left\{ \frac{m_\pi}{f_\pi^2} \right\} + \left\{ -\frac{m_\pi^2(24c_0 + 4c_1 - 3C_2 - 6C_3 - 6c_4)}{6f_\pi^2} \right\} + \left\{ \left[ \frac{m_\pi^2(8d_1 m_\pi + d_2 \delta_2 + 2d_3 \delta_2)}{2f_\pi^2 M_0^2} \right] + [-\frac{1}{8}V(m_K^2, m_\pi) \right. \right. \\ &\quad \left. \left. - \frac{1}{4}V(m_\pi^2, m_\pi) - \frac{1}{9}m_\pi^2 W_2(m_\eta) - m_\pi^2 W_2(m_\pi) + 4m_\pi^2 Y_2(m_\pi)] \right\}, \\ T_{\pi\Sigma_c}^{(2)} &= \left\{ -\frac{m_\pi}{f_\pi^2} \right\} + \left\{ -\frac{m_\pi^2(24c_0 + 4c_1 - 3C_2 - 6c_4)}{6f_\pi^2} \right\} + \left\{ \left[ -\frac{m_\pi^2(8d_1 m_\pi - d_2 \delta_2)}{2f_\pi^2 M_0^2} \right] + [-\frac{1}{8}V(m_K^2, -m_\pi) \right. \right. \\ &\quad \left. \left. - \frac{1}{2}V(m_\pi^2, -m_\pi) - \frac{1}{4}V(m_\pi^2, m_\pi) - \frac{1}{9}m_\pi^2 W_2(m_\eta) + m_\pi^2 W_2(m_\pi)] \right\}, \\ T_{K\Omega_c}^{(1/2)} &= \left\{ \frac{m_K}{f_K^2} \right\} + \left\{ -\frac{m_K^2(24c_0 + 4c_1 - 3C_2 - 6c_4)}{6f_K^2} \right\} + \left\{ \left[ \frac{m_K^2(8d_1 m_K + d_2 \delta_2)}{2f_K^2 M_0^2} \right] + [-\frac{1}{4}V(m_K^2, -m_K) \right. \right. \\ &\quad \left. \left. - \frac{1}{4}V(m_K^2, m_K) - \frac{3}{16}V(m_\eta^2, m_K) - \frac{3}{16}V(m_\pi^2, m_K) + \frac{8}{9}m_K^2 W_2(m_\eta)] \right\}, \\ T_{K\Xi_c}^{(0)} &= \left\{ \frac{m_K}{f_K^2} \right\} + \left\{ -\frac{m_K^2(24c_0 - 8c_1 + 3C_3 - 6c_4)}{6f_K^2} \right\} + \left\{ \left[ \frac{m_K^2(8d_1 m_K - d_3 \delta_2)}{2f_K^2 M_0^2} \right] + [-\frac{1}{16}V(m_K^2, -m_K) \right. \right. \\ &\quad \left. \left. - \frac{1}{4}V(m_K^2, m_K) + \frac{3}{16}V(m_\eta^2, -m_K) - \frac{3}{16}V(m_\pi^2, -m_K) - \frac{3}{16}V(m_\pi^2, m_K) + \frac{1}{18}m_K^2 W_2(m_\eta) \right. \right. \\ &\quad \left. \left. + m_K^2 Y_2(m_\eta) - 3P_2 + \frac{U_2}{2}] \right\}, \\ T_{K\Xi_c}^{(1)} &= \{0\} + \left\{ -\frac{m_K^2(24c_0 + 4c_1 - 3C_2 - 3C_3 - 6c_4)}{6f_K^2} \right\} + \left\{ \left[ \frac{m_K^2 \delta_2(d_2 + d_3)}{2f_K^2 M_0^2} \right] + [-\frac{1}{16}V(m_K^2, -m_K) \right. \right. \\ &\quad \left. \left. - \frac{3}{16}V(m_\eta^2, -m_K) - \frac{3}{16}V(m_\eta^2, m_K) - \frac{1}{16}V(m_\pi^2, -m_K) - \frac{1}{8}V(m_\pi^2, m_K) + \frac{1}{18}m_K^2 W_2(m_\eta) \right. \right. \\ &\quad \left. \left. + m_K^2 Y_2(m_\eta) + P_2 - \frac{U_2}{6}] \right\}, \end{aligned}$$

$$\begin{aligned}
T_{K\Sigma_c}^{(1/2)} &= \left\{ \frac{m_K}{2f_K^2} \right\} + \left\{ -\frac{m_K^2(48c_0 - 28c_1 + 3C_2 - 12c_4)}{12f_K^2} \right\} + \left\{ \left[ \frac{m_K^2(8d_1m_K - d_2\delta_2)}{4f_K^2M_0^2} \right] + \left[ \frac{3}{16}V(m_K^2, -m_K) \right. \right. \\
&\quad \left. \left. - \frac{1}{16}V(m_K^2, m_K) + \frac{3}{32}V(m_\eta^2, -m_K) - \frac{5}{32}V(m_\pi^2, -m_K) + \frac{2}{9}m_K^2W_2(m_\eta) - \frac{4U_2}{3} \right] \right\}, \\
T_{K\Sigma_c}^{(3/2)} &= \left\{ -\frac{m_K}{f_K^2} \right\} + \left\{ -\frac{m_K^2(24c_0 + 4c_1 - 3C_2 - 6c_4)}{6f_K^2} \right\} + \left\{ \left[ -\frac{m_K^2(8d_1m_K - d_2\delta_2)}{2f_K^2M_0^2} \right] + \left[ -\frac{3}{8}V(m_K^2, -m_K) \right. \right. \\
&\quad \left. \left. - \frac{1}{4}V(m_K^2, m_K) - \frac{3}{16}V(m_\eta^2, -m_K) - \frac{1}{16}V(m_\pi^2, -m_K) + \frac{2}{9}m_K^2W_2(m_\eta) + \frac{2U_2}{3} \right] \right\}, \\
T_{\eta\Omega_c}^{(0)} &= \{0\} + \left\{ -\frac{12c_0m_\eta^2 + 8c_1m_\eta^2 - 4c_1m_\pi^2 - 2C_2m_\eta^2 - 2C_3m_\eta^2 - 3c_4m_\eta^2}{3f_\eta^2} \right\} + \left\{ \left[ \frac{2m_\eta^2\delta_2(d_2 + d_3)}{3f_\eta^2M_0^2} \right] + \left[ \right. \right. \\
&\quad \left. \left. - \frac{3}{8}V(m_K^2, -m_\eta) - \frac{3}{8}V(m_K^2, m_\eta) - \frac{64}{27}m_K^2W_2(m_\eta) + \frac{4}{3}m_K^2W_2(m_K) + \frac{8}{3}m_K^2Y_2(m_K) + \frac{28}{27}m_\pi^2W_2(m_\eta) \right] \right\}, \\
T_{\eta\Xi_c}^{(1/2)} &= \{0\} + \left\{ -\frac{48c_0m_\eta^2 + 8c_1m_\eta^2 - 4c_1m_\pi^2 - 5C_2m_\eta^2 + 4C_3m_\eta^2 - 12c_4m_\eta^2}{12f_\eta^2} \right\} + \left\{ \left[ \frac{m_\eta^2\delta_2(5d_2 - 4d_3)}{12f_\eta^2M_0^2} \right] + \left[ \right. \right. \\
&\quad \left. \left. - \frac{15}{32}V(m_K^2, -m_\eta) - \frac{15}{32}V(m_K^2, m_\eta) - \frac{4}{27}m_K^2W_2(m_\eta) - \frac{8}{3}m_K^2Y_2(m_\eta) + \frac{5}{3}m_K^2W_2(m_K) + \frac{2}{3}m_K^2Y_2(m_K) \right. \right. \\
&\quad \left. \left. + \frac{7}{108}m_\pi^2W_2(m_\eta) + \frac{7}{6}m_\pi^2Y_2(m_\eta) - \frac{1}{4}m_\pi^2W_2(m_\pi) - \frac{1}{2}m_\pi^2Y_2(m_\pi) \right] \right\}, \\
T_{\eta\Xi_c}^{(1)} &= \{0\} + \left\{ -\frac{24c_0m_\eta^2 - 8c_1m_\eta^2 + 4c_1m_\pi^2 - C_2m_\eta^2 - C_3m_\eta^2 - 6c_4m_\eta^2}{6f_\eta^2} \right\} + \left\{ \left[ \frac{m_\eta^2\delta_2(d_2 + d_3)}{6f_\eta^2M_0^2} \right] + \left[ \right. \right. \\
&\quad \left. \left. - \frac{3}{16}V(m_K^2, -m_\eta) - \frac{3}{16}V(m_K^2, m_\eta) - \frac{16}{27}m_K^2W_2(m_\eta) + \frac{2}{3}m_K^2W_2(m_K) + \frac{4}{3}m_K^2Y_2(m_K) + \frac{7}{27}m_\pi^2W_2(m_\eta) \right. \right. \\
&\quad \left. \left. - \frac{2}{3}m_\pi^2W_2(m_\pi) - \frac{2}{3}m_\pi^2Y_2(m_\pi) \right] \right\}, \tag{13}
\end{aligned}$$

where

$$C_2 = c_2 - \frac{2g_2^2 + g_1^2}{4M_0}, \quad C_3 = c_3 + \frac{2g_2^2 - g_1^2}{4M_0}, \quad d_1 = \frac{g_1^2 + 2g_2^2}{64}, \quad d_2 = \frac{g_1^2 + g_2^2}{4}, \quad d_3 = \frac{g_1^2 - g_2^2}{4}. \tag{14}$$

Moreover, with crossing symmetry we get

$$\begin{aligned}
T_{K\Sigma_c}^{(3/2)} &= \frac{1}{3} [T_{K\Sigma_c}^{(3/2)} + 2T_{K\Sigma_c}^{(1/2)}]_{m_K \rightarrow -m_K}, \quad T_{K\Sigma_c}^{(1/2)} = \frac{1}{3} [4T_{K\Sigma_c}^{(3/2)} - T_{K\Sigma_c}^{(1/2)}]_{m_K \rightarrow -m_K}, \quad T_{K\Omega_c}^{(1/2)} = [T_{K\Omega_c}^{(1/2)}]_{m_K \rightarrow -m_K}, \\
T_{K\Xi_c}^{(1)} &= \frac{1}{2} [T_{K\Xi_c}^{(1)} + T_{K\Xi_c}^{(0)}]_{m_K \rightarrow -m_K}, \quad T_{K\Xi_c}^{(0)} = \frac{1}{2} [3T_{K\Xi_c}^{(1)} - T_{K\Xi_c}^{(0)}]_{m_K \rightarrow -m_K}. \tag{15}
\end{aligned}$$

### C. $\phi B_6^*$ scattering

There are also 19 independent  $T$ -matrices for the scattering between pseudoscalar meson and the excited charmed sextet baryon. Even including the loop correction, we notice that  $T_{\phi B_6^*}^{(I)}$  is the same as  $T_{\phi B_6}^{(I)}$  in the heavy baryon symmetry limit. Taking into account of the heavy baryon symmetry breaking effect, we obtain  $T_{\phi B_6^*}^{(I)}$  after making the following replacements in the expressions of the corresponding  $T_{\phi B_6}^{(I)}$  in Eqs (13,15)

$$\begin{aligned}
c_1 &\rightarrow \tilde{c}_1, \quad C_2 \rightarrow \tilde{c}_2 - \frac{g_5^2}{4M_0}, \quad C_3 \rightarrow \tilde{c}_3 - \frac{g_5^2}{4M_0}, \quad c_4 \rightarrow \tilde{c}_4, \quad d_1 \rightarrow \frac{g_5^2}{64}, \quad d_2 \rightarrow \frac{g_5^2}{4}, \quad d_3 \rightarrow \frac{g_5^2}{4}, \\
P_2 &\rightarrow P_3, \quad U_2 \rightarrow U_3, \quad W_2(m_X) \rightarrow W_3(m_X), \quad Y_2(m_X) \rightarrow Y_3(m_X). \tag{16}
\end{aligned}$$

We have listed the  $T$ -matrices of the pseudoscalar meson and charmed baryon scattering in the above three subsections. We have assumed the SU(3) flavor symmetry and taken the SU(3) breaking effect into account perturbatively. One can also study the scattering of  $\pi B_3$ ,  $\pi B_6$ , and  $\pi B_6^*$  with SU(2) flavor symmetry. We can construct the relevant Lagrangians with exact SU(2) chiral symmetry from the beginning.

Alternatively, we can extract SU(2) Lagrangians from the SU(3) Lagrangians in Eqs. (1), (2), (6), (7), and (8). Now the coupling constants  $g_i$  and other LECs are different from those in the SU(3) case. Then we can obtain the SU(2)  $T$ -matrices from the SU(3) ones. More specially, we may drop the terms proportional to  $V(m_K^2, \pm m_\pi)$ ,  $Y_i(m_\eta)$ , and  $W_i(m_\eta)$  in the SU(3)  $T_{\pi B}$ , and replace  $g_1$  and other LECs with new independent ones  $g_{1\Sigma_c}$ ,  $g_{1\Xi_c}$ ,  $g_{1\Omega_c}$  etc. After that we get the SU(2)  $T_{\pi B}$ . The contributions of the dropped terms actually are absorbed by the redefined LECs at  $O(\epsilon^3)$ .

Unfortunately, the investigation of the  $\pi B$  scattering with SU(2) chiral perturbation theory introduces more independent LECs. Especially the SU(2) LECs at  $O(\epsilon^3)$  can not be neglected. We do not include the contribution from the kaon and eta explicitly, which contribute to  $O(\epsilon^3)$  LECs here. In the following we will concentrate on the SU(3) case only.

### III. LOW ENERGY CONSTANTS

Similar to the nucleon case [26, 27], the chiral correction to the charmed baryon axial-vector coupling would also be  $O(\epsilon^2)$ , which contributes to the  $T$ -matrices at  $O(\epsilon^4)$  or higher order, thus can be neglected. Using  $|g_2| = 0.60$  and  $|g_4| = 1.0$  obtained by fitting the decay widths of  $\Sigma_c$  and  $\Sigma_c^*$  [5],  $|g_1| = \sqrt{8/3}|g_2|$  with the quark model symmetry, and  $|g_3| = \sqrt{3}/2|g_1|$ ,  $|g_5| = 3/2|g_1|$ ,  $|g_6| = 0$  with heavy quark spin symmetry, we have

$$|g_1| = 0.98, \quad |g_2| = 0.60, \quad |g_3| = 0.85, \quad |g_4| = 1.0, \quad |g_5| = 1.5, \quad |g_6| = 0. \quad (17)$$

We also need [28–30]

$$m_\pi = 140 \text{ MeV}, \quad m_K = 494 \text{ MeV}, \quad f_\pi = 92 \text{ MeV}, \quad f_K = 113 \text{ MeV}, \quad f_\eta = 1.2 f_K. \quad (18)$$

Since there are no available experimental data to extract the low energy constants at  $O(\epsilon^2)$ , we utilize the crude SU(4) flavor symmetry to make a rough estimate of some of these LECs in Appendix B,

$$\begin{aligned} \bar{c}_0 &= -0.39 \text{ GeV}, \quad \bar{c}_1 = -0.52 \text{ GeV}, \quad \bar{c}_2 = -1.78 \text{ GeV} + \frac{1}{3} \frac{\alpha'}{4\pi f}, \quad \bar{c}_3 = -0.03 \text{ GeV} - \frac{1}{3} \frac{\alpha'}{4\pi f}, \\ c_0 &= -0.79 \text{ GeV}, \quad c_1 = -0.98 \text{ GeV}, \quad c_2 = -2.07 \text{ GeV} - 2 \frac{\alpha'}{4\pi f}, \quad c_3 = \frac{\alpha'}{4\pi f}, \quad c_4 = -0.84 \text{ GeV}. \end{aligned} \quad (19)$$

We would assume  $\tilde{c}_i = c_i$  with the heavy quark spin symmetry in the numerical calculation. As for the dimensionless LEC  $\alpha'$ , we will take it to be in the natural range of  $[-1, 1]$  as in Ref. [18].

### IV. NUMERICAL RESULTS AND DISCUSSIONS

We list the  $T$ -matrices order by order for the pseudoscalar meson and charmed baryon scattering in Tables I, II, III. The positive real parts of the scattering lengths indicate that there exists the attractive interaction in the following channels:  $\pi\Lambda_c^{(1)}$ ,  $\pi\Xi_c^{(1/2)}$ ,  $K\Xi_c^{(0)}$ ,  $K\Xi_c^{(1)}$ ,  $\bar{K}\Lambda_c^{(1/2)}$ ,  $\bar{K}\Xi_c^{(0)}$ ,  $\eta\Lambda_c^{(0)}$ ,  $\eta\Xi_c^{(1/2)}$ ,  $\pi\Xi_c'^{(1/2)}$ ,  $\pi\Sigma_c^{(0)}$ ,  $\pi\Sigma_c^{(1)}$ ,  $K\Omega_c^{(1/2)}$ ,  $K\Xi_c'^{(0)}$ ,  $K\Xi_c'^{(1)}$ ,  $K\Sigma_c^{(1/2)}$ ,  $\bar{K}\Xi_c'^{(0)}$ ,  $\bar{K}\Sigma_c^{(1/2)}$ ,  $\bar{K}\Sigma_c^{(3/2)}$ ,  $\eta\Omega_c^{(0)}$ ,  $\eta\Xi_c'^{(1/2)}$ ,  $\eta\Sigma_c^{(1)}$ ,  $\pi\Xi_c^{*(1/2)}$ ,  $\pi\Sigma_c^{*(0)}$ ,  $\pi\Sigma_c^{*(1)}$ ,  $K\Omega_c^{*(1/2)}$ ,  $K\Xi_c^{*(0)}$ ,  $K\Xi_c^{*(1)}$ ,  $K\Sigma_c^{*(1/2)}$ ,  $\bar{K}\Xi_c^{*(0)}$ ,  $\bar{K}\Sigma_c^{*(1/2)}$ ,  $\bar{K}\Sigma_c^{*(3/2)}$ ,  $\eta\Omega_c^{*(0)}$ ,  $\eta\Xi_c^{*(1/2)}$ , and  $\eta\Sigma_c^{*(1)}$ , where the superscripts refer to the isospin.

There is an undetermined constant  $\alpha'$  at  $O(\epsilon^2)$ . We allow  $\alpha'$  to vary from -1 to 1. Its contribution is small. The variation of the scattering length is less than one fifth of the central value in almost 40 channels among the total 49 channels.

From the tables, the leading order contribution from the chiral connection dominates the total  $T$ -matrices for the most  $\pi B$  channels. We regard one scattering channel as convergent when

$$\left\{ \begin{array}{ll} |T_{\phi B}|_{O(\epsilon^3)} < \frac{1}{2} |T_{\phi B}|_{O(\epsilon^2)} < \frac{1}{4} |T_{\phi B}|_{O(\epsilon^1)}, & |T_{\phi B}|_{O(\epsilon^1)} \neq 0 \\ |T_{\phi B}|_{O(\epsilon^3)} < \frac{1}{2} |T_{\phi B}|_{O(\epsilon^2)}, & |T_{\phi B}|_{O(\epsilon^1)} = 0 \end{array} \right. \quad (20)$$

With the above criteria there exist twelve convergent channels:  $T_{\pi\Lambda_c}^{(1)}$ ,  $T_{\pi\Xi_c}^{(1/2)}$ ,  $T_{\eta\Lambda_c}^{(0)}$ ,  $T_{\eta\Xi_c}^{(1/2)}$ ,  $T_{\pi\Sigma_c}^{(1)}$ ,  $T_{\eta\Omega_c}^{(0)}$ ,  $T_{\eta\Xi_c'}^{(1/2)}$ ,  $T_{\eta\Sigma_c}^{(1)}$ ,  $T_{\pi\Sigma_c^*}^{(1)}$ ,  $T_{\eta\Xi_c^*}^{(1/2)}$ ,  $T_{\eta\Sigma_c^*}^{(1)}$ , and  $T_{\eta\Xi_c^*}^{(1/2)}$ . The scattering lengths of the above channels are positive. In other words, the interaction between the pseudoscalar meson and heavy baryon is attractive. The chiral expansion of the  $KB$  channels converges badly mainly due to the large mass of kaon.



For the eta meson scattering off the charmed baryon, the loop diagrams in Fig. 1(I) do not contribute to the real part of the  $T$ -matrix at the threshold as can be seen from Eqs. (10,13,A1,A2). Only the loop diagrams in Fig. 1(II) contribute to the real part of the  $T_{B\eta}$ -matrix, which is helpful to the convergence in the  $\eta$  channel.

At present there is not enough experimental information on the pseudoscalar meson and heavy baryon scattering. We are unable to determine the low energy constants at  $O(\epsilon^3)$ . With the very crude nonanalytic dominance approximation, we study the convergence of the chiral expansion further in Appendix C. Under this approximation, the convergence becomes better in the most channels, especially in  $\bar{K}\Lambda_c^{(1/2)}$ ,  $K\Omega_c^{(1/2)}$ ,  $K\Sigma_c^{(3/2)}$ ,  $\bar{K}\Omega_c^{(1/2)}$ ,  $\bar{K}\Sigma_c^{(1/2)}$ ,  $K\Omega_c^{*(1/2)}$ ,  $K\Sigma_c^{*(3/2)}$ ,  $\bar{K}\Omega_c^{*(1/2)}$ , and  $\bar{K}\Sigma_c^{*(1/2)}$ .

In order to check where the large correction at  $O(\epsilon^3)$  comes from, we separate the different contributions to  $T_{\phi B}^{(I)}$  at  $O(\epsilon^3)$  in natural units of  $m_\phi/f_\phi^2$  in Table IV. We notice that the tree contribution at  $O(\epsilon^3)$  is really small since the recoil correction should be suppressed for a heavy charmed baryon. The inclusion of the excited charmed sextet does not suppress the loop correction for the channels of the ground charmed baryons.

It is interesting to notice that the inclusion of the  $B_6^*$  intermediate states does not make the convergence better. Let's denote the contribution of the intermediate particle  $X$  to the  $T$ -matrix through the axial couplings in the heavy quark symmetry limit as  $C_X$ . For the  $B_3\phi$  scattering, we can get the following ratio from Eqs. (10,A1)

$$\left. \frac{C_{B_6}}{C_{B_6^*}} \right|_{\text{for } B_3\phi \text{ scattering}} = \frac{3g_2^2}{2g_4^2} = \frac{1}{2}. \quad (21)$$

For the  $B_6\phi$  scattering,

$$\left. \frac{C_{B_6}}{C_{B_6^*}} \right|_{\text{for } B_6\phi \text{ scattering}} = \frac{3g_1^2}{2g_3^2} = 2. \quad (22)$$

And for the  $B_6^*\phi$  scattering, the ratio is

$$\left. \frac{C_{B_6}}{C_{B_6^*}} \right|_{\text{for } B_6^*\phi \text{ scattering}} = \frac{3g_3^2}{5g_5^2} = \frac{1}{5}. \quad (23)$$

One notices that  $C_{B_6^*}$  is larger than  $C_{B_6}$  for the  $B_3\phi$  and  $B_6^*\phi$  scattering, while it is smaller than  $C_{B_6}$  for the  $B_6\phi$  scattering. The correction from the  $B_6^*$  and  $B_6$  states has the same sign as required from heavy quark symmetry. Their contribution is constructive, which worsens the convergence.

From Tables II and III, one notices that the numerical value of  $T_{\phi B_6}^I$  is very close to that of the corresponding  $T_{\phi B_6^*}^I$  at every order. As can be seen in Table IV, the contribution of the sum of all the loop diagrams to  $T_{\phi B_6}^I$  and  $T_{\phi B_6^*}^I$  is almost the same, which is the manifestation of the heavy flavor symmetry.

Sometimes the nearby resonances or possible molecular states in the pseudoscalar meson and heavy baryon scattering channel might also destroy the convergence of the chiral expansion. For example, the  $1/2^-(3/2^-)$  charmed baryons couple strongly to the Goldstone boson and  $1/2^+(3/2^+)$  charmed baryons based on a unitary baryon-meson coupled-channel model in Ref. [31]. The convergence of the chiral expansion might improve if the  $1/2^-$  and  $3/2^-$  charmed baryons are included explicitly.

One may also wonder whether the recoil correction might spoil the convergence. In the past several years there has been some progress in the development of the  $\chi$ PT in the covariant form such as the extended-on-mass-shell renormalization scheme [15, 32, 33] and infrared regularization method [34]. It will be very interesting to compare the results within the different schemes.

We estimate the LECs at  $O(\epsilon^2)$  assuming the SU(4) flavor symmetry and using the pseudoscalar meson and nucleon-octet coupling constants as input. However, the SU(4) flavor symmetry is broken in nature. The convergence of the chiral expansion might improve if the LECs could be determined more accurately.

## V. CONCLUSIONS

In this work, we have studied the pseudoscalar meson and charmed baryon scattering length to  $O(\epsilon^3)$  with HB $\chi$ PT. The convergence of the chiral expansion of some pion and eta channels is good. Because of the large heavy baryon mass, the recoil correction is small.

It is easy to get the  $T$ -matrices for the pseudoscalar meson and bottomed baryon scattering from those in Sec. II with the corresponding parameters replaced. The numerical results do not change much due to the heavy quark flavor symmetry.



According to our convention, the scattering length with a positive real part indicates there is attraction in this channel, which provides useful information on the strong interaction between the pseudoscalar meson and heavy flavor baryon. For example, one may have a rough idea in which channels there may (or not) exist loosely bound molecular states composed of a heavy flavor baryon and a pseudoscalar meson. These systems are similar to the pionic hydrogen. Moreover, we hope our present calculation, especially nonanalytic parts, would be useful to the chiral extrapolation of future simulation of the pseudoscalar meson and heavy baryon scattering on the lattice.

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### Appendix A: Some functions and constants in the $T$ -matrices

We list the functions and constants in the  $T$ -matrices here,

$$\begin{aligned}
P_1 &= -\frac{3g_6^2 m_K^2 H_2(0, m_\eta, m_\pi)}{4}, & P_2 &= -\frac{3g_2^2 m_K^2 H_2(-\delta_2, m_\eta, m_\pi)}{4}, & P_3 &= -\frac{g_4^2 m_K^2 H_2(-\delta_3, m_\eta, m_\pi)}{4}, \\
U_1 &= -\frac{m_K^2 \{3g_2^2 H_2(\delta_2, m_\eta, m_\pi) + 2g_4^2 H_2(\delta_3, m_\eta, m_\pi)\}}{4}, & U_2 &= -\frac{m_K^2 \{3g_1^2 H_2(0, m_\eta, m_\pi) + 2g_3^2 H_2(\delta_1, m_\eta, m_\pi)\}}{4}, \\
U_3 &= -\frac{m_K^2 \{5g_5^2 H_2(0, m_\eta, m_\pi) + 3g_3^2 H_2(-\delta_1, m_\eta, m_\pi)\}}{12}, & W_1(m) &= \frac{-3g_2^2 H_2(\delta_2, m, m) - 2g_4^2 H_2(\delta_3, m, m)}{4}, \\
W_2(m) &= \frac{-3g_1^2 H_2(0, m, m) - 2g_3^2 H_2(\delta_1, m, m)}{4}, & W_3(m) &= \frac{-5g_5^2 H_2(0, m, m) - 3g_3^2 H_2(-\delta_1, m, m)}{12}, \\
Y_1(m) &= -\frac{3g_6^2 H_2(0, m, m)}{4}, & Y_2(m) &= -\frac{3g_2^2 H_2(-\delta_2, m, m)}{4}, & Y_3(m) &= -\frac{g_4^2 H_2(-\delta_3, m, m)}{4}, \tag{A1}
\end{aligned}$$

$$V(m^2, \omega) = \left[ -\frac{\omega^3}{2\pi^2 f^4} \right] + \frac{\omega^3 \ln \frac{|m|}{\lambda}}{\pi^2 f^4} - \frac{\omega^2}{\pi^2 f^4} \begin{cases} -\sqrt{m^2 - \omega^2} \arccos\left(-\frac{\omega}{|m|}\right) & m^2 \geq \omega^2 \\ \sqrt{\omega^2 - m^2} \ln \frac{\sqrt{\omega^2 - m^2} - \omega}{|m|} & m^2 < \omega^2, \omega < 0, \\ \sqrt{\omega^2 - m^2} \left( -\ln \frac{\sqrt{\omega^2 - m^2} + \omega}{|m|} + i\pi \right) & m^2 < \omega^2, \omega \geq 0 \end{cases} \tag{A2}$$

where

$$\begin{aligned}
H_1(m^2, \omega) &= \left[ \frac{6m^2 \omega - 5\omega^3}{72\pi^2} \right] + \frac{2\omega^3 - 3m^2 \omega}{24\pi^2} \ln \frac{|m|}{\lambda} + \frac{m^2 - \omega^2}{12\pi^2} \begin{cases} -\sqrt{m^2 - \omega^2} \arccos \frac{\omega}{|m|} & m^2 > \omega^2 \\ \sqrt{\omega^2 - m^2} \ln \frac{-\omega + \sqrt{\omega^2 - m^2}}{|m|} & m^2 < \omega^2, \omega < 0, \\ \sqrt{\omega^2 - m^2} (i\pi - \ln \frac{\omega + \sqrt{\omega^2 - m^2}}{|m|}) & m^2 < \omega^2, \omega > 0 \end{cases} \\
H_2(\omega, m, M) &= \frac{1}{f^4} \begin{cases} \frac{H_1(m^2, -\omega) - H_1(M^2, -\omega)}{m^2 - M^2} & m^2 \neq M^2 \\ \frac{\partial H_1(z, -\omega)}{\partial z} \Big|_{\{z \rightarrow m^2\}} & m^2 = M^2 \end{cases} \tag{A3}
\end{aligned}$$

### Appendix B: Determination of Some LECs with SU(4) Flavor Symmetry

The SU(4) flavor 20'-plet includes the nucleon octet,  $\Lambda_c$  triplet,  $\Sigma_c$  sextet, and  $\Xi_{cc}$  triplet, which can be expressed by a 3-rank tensor  $T_{abc}$ :

$$T_{cba} = -T_{abc}, \quad T_{cab} = -T_{abc} + T_{acb}. \quad (a, b, c = 1, 2, 3, 4 \text{ are the flavor labels.}) \tag{B1}$$

With the SU(4) 20'-representation and the isospin, hypercharge, and charm of the physical particles, one obtains the individual components

$$\begin{aligned}
T_{112} &= \frac{1}{\sqrt{2}} P, & T_{122} &= \frac{1}{\sqrt{2}} N, & T_{113} &= -\frac{1}{\sqrt{2}} \Sigma^+, & T_{223} &= \frac{1}{\sqrt{2}} \Sigma^-, & T_{133} &= -\frac{1}{\sqrt{2}} \Xi^0, & T_{233} &= \frac{1}{\sqrt{2}} \Xi^-, \\
T_{132} &= -\frac{1}{\sqrt{2}} \Lambda, & T_{123} &= \frac{1}{\sqrt{2}} \Sigma^0 - \frac{1}{2\sqrt{3}} \Lambda, & T_{114} &= \frac{1}{\sqrt{2}} \Sigma_c^{++}, & T_{224} &= \frac{1}{\sqrt{2}} \Sigma_c^0, & T_{334} &= \frac{1}{\sqrt{2}} \Omega_c^0, & T_{144} &= \frac{1}{\sqrt{2}} \Sigma_{cc}^{++},
\end{aligned}$$

$$\begin{aligned}
T_{244} &= \frac{1}{\sqrt{2}}\Sigma_{cc}^+, & T_{344} &= \frac{1}{\sqrt{2}}\Omega_{cc}^+, & T_{142} &= \frac{1}{\sqrt{3}}\Lambda_c^+, & T_{124} &= \frac{1}{2}\Sigma_c^+ + \frac{1}{2\sqrt{3}}\Lambda_c^+, & T_{143} &= \frac{1}{\sqrt{3}}\Xi_c^+, \\
T_{134} &= \frac{1}{2}\Xi_c'^+ + \frac{1}{2\sqrt{3}}\Xi_c^+, & T_{243} &= \frac{1}{\sqrt{3}}\Xi_c^0, & T_{234} &= \frac{1}{2}\Xi_c'^0 + \frac{1}{2\sqrt{3}}\Xi_c^0.
\end{aligned} \tag{B2}$$

where we have normalized  $T_{abc}$  so that the Lagrangian of the self-energy is  $\bar{T}^{abc}(iv \cdot D)T_{abc}$ . With the chiral symmetry, parity, C-parity, and Hermiticity, we can construct the Lagrangian

$$\begin{aligned}
\mathcal{L}_{\text{SU}(4)}^{(2)} &= \alpha_0 \bar{T} T \text{Tr}[\chi_+] + \alpha_1 \bar{T}^{abc} \tilde{\chi}_{+a} {}^d T_{dbc} + \alpha_2 \bar{T}^{abc} \tilde{\chi}_{+a} {}^d T_{dcb} + \alpha_3 \bar{T} T \text{Tr}[v \cdot u \ v \cdot u] + \alpha_4 \bar{T}^{ibc} v \cdot u_i {}^n v \cdot u_n {}^j T_{jbc} \\
&\quad + \alpha_5 \bar{T}^{ibc} v \cdot u_i {}^n v \cdot u_n {}^j T_{jcb} + \alpha_6 \bar{T}^{ija} v \cdot u_i {}^m v \cdot u_j {}^n T_{mna} + \alpha_7 \bar{T}^{ija} v \cdot u_i {}^m v \cdot u_j {}^n T_{nma},
\end{aligned} \tag{B3}$$

where  $u_\mu$  and  $\chi_+$  are similar to those in Eq. (5) with the extended  $\chi^{\text{ext}} = \text{diag}(m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2, 2m_D^2 - m_\pi^2)$ , and

$$\phi^{\text{ext}} = \sqrt{2} \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta_c}{\sqrt{12}} & \pi^+ & K^+ & \bar{D}^0 \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta_c}{\sqrt{12}} & K^0 & D^- \\ K^- & \frac{\bar{K}^0}{\sqrt{2}} & -\frac{2}{\sqrt{6}}\eta + \frac{\eta_c}{\sqrt{12}} & D_s^- \\ D^0 & D^+ & D_s^+ & \frac{-3\eta_c}{\sqrt{12}} \end{pmatrix}. \tag{B4}$$

$\mathcal{L}_{\text{SU}(4)}^{(2)}$  contains  $\mathcal{L}_{B_8\phi}^{(2)}$  for the nucleon octet,  $\mathcal{L}_{B_3\phi}^{(2)}$  for the  $\Lambda_c$  triplet,  $\mathcal{L}_{B_6\phi}^{(2)}$  for the  $\Sigma_c$  triplet, and so on. So comparing  $\mathcal{L}_{B_8\phi}^{(2)}$  with that in Ref. [23] and using their values of LECs  $b_0, b_D, b_F, d_0, d_1, d_D, d_F$ , we get

$$\begin{aligned}
\alpha_0 &= b_0 + \frac{2}{3}b_D = -0.79 \text{ GeV}, & \alpha_1 &= 4b_F = -1.96 \text{ GeV}, & \alpha_2 &= -2b_D - 2b_F = 0.89 \text{ GeV}, & \alpha_3 &= \frac{\alpha'}{4\pi f}, \\
\alpha_4 &= 8d_0 + 4d_1 + 8d_D - 4\frac{\alpha'}{4\pi f} = -4.13 \text{ GeV} - 4\frac{\alpha'}{4\pi f}, & \alpha_5 &= -4d_1 - 4d_D - 4d_F = -0.19 \text{ GeV}, \\
\alpha_6 &= 8d_0 + 8d_D - 8d_F - 4\frac{\alpha'}{4\pi f} = -0.67 \text{ GeV} - 4\frac{\alpha'}{4\pi f}, \\
\alpha_7 &= -8d_0 - 4d_1 - 8d_D + 8d_F + 4\frac{\alpha'}{4\pi f} = -1.00 \text{ GeV} + 4\frac{\alpha'}{4\pi f},
\end{aligned} \tag{B5}$$

with a still unknown dimensionless constant  $\alpha'$ . Then comparing  $\mathcal{L}_{B_3\phi}^{(2)}$  and  $\mathcal{L}_{B_6\phi}^{(2)}$  with Eq. (6), one gets

$$\begin{aligned}
\bar{c}_0 &= \frac{1}{2}\alpha_0 = -0.39 \text{ GeV}, & \bar{c}_1 &= \frac{5}{12}\alpha_1 + \frac{1}{3}\alpha_2 = -0.52 \text{ GeV}, & \bar{c}_2 &= 2\alpha_3 + \frac{5}{12}\alpha_4 + \frac{1}{3}\alpha_5 = -1.78 \text{ GeV} + \frac{1}{3}\frac{\alpha'}{4\pi f}, \\
\bar{c}_3 &= -\alpha_3 - \frac{1}{12}\alpha_6 + \frac{1}{12}\alpha_7 = -0.03 \text{ GeV} - \frac{1}{3}\frac{\alpha'}{4\pi f}, & c_0 &= \alpha_0 = -0.79 \text{ GeV}, & c_1 &= \frac{1}{2}\alpha_1 = -0.98 \text{ GeV}, \\
c_2 &= \frac{1}{2}\alpha_4 = -2.07 \text{ GeV} - 2\frac{\alpha'}{4\pi f}, & c_3 &= \alpha_3 = \frac{\alpha'}{4\pi f}, & c_4 &= \frac{1}{2}\alpha_6 + \frac{1}{2}\alpha_7 = -0.84 \text{ GeV}.
\end{aligned} \tag{B6}$$

### Appendix C: Nonanalytic dominance approximation and the convergence of the chiral expansion

The analytic terms from loop corrections are the polynomials of  $\epsilon$  possessing the symmetries of Lagrangians. They can be absorbed by the LECs. In other words, the tree and partial loop corrections have the same chiral structure. One may divide the  $T$ -matrix into the analytic and nonanalytic part. The analytic contribution originates from both loop and tree diagrams, while the nonanalytic contribution originates only from the loop graphs. One may also use the nonanalytic part to discuss the convergence of the  $T$ -matrix since the LECs of the third order can not be determined now.

For the  $\pi\pi$  scattering length, the ratio of the analytic contribution  $a_{\pi\pi}^{\text{anal},I}$  to the nonanalytic contribution  $a_{\pi\pi}^{L,I}$  is small[35],

$$\left. \frac{a_{\pi\pi}^{\text{anal},0}}{a_{\pi\pi}^{L,0}} \right|_{O(p^4)} = \frac{0.005}{0.039}, \quad \left. \frac{a_{\pi\pi}^{\text{anal},0}}{a_{\pi\pi}^{L,0}} \right|_{O(p^6)} = \frac{0.001}{0.016}, \quad \left. \frac{a_{\pi\pi}^{\text{anal},0} - a_{\pi\pi}^{\text{anal},2}}{a_{\pi\pi}^{L,0} - a_{\pi\pi}^{L,2}} \right|_{O(p^4)} = \frac{0.006}{0.036}, \quad \left. \frac{a_{\pi\pi}^{\text{anal},0} - a_{\pi\pi}^{\text{anal},2}}{a_{\pi\pi}^{L,0} - a_{\pi\pi}^{L,2}} \right|_{O(p^6)} = \frac{0.001}{0.015}.$$

Because of lack of enough data, we are still unable to estimate the LECs at  $O(\epsilon^3)$  accurately. As a very crude approximation, we invoke the “nonanalytic dominance approximation” to check the convergence of the chiral expansion, which assumes large cancellation of the analytic terms between loop and tree diagrams. Under this approximation, we list the scattering lengths with the nonanalytic approximation in the last column of Tables I, II, III. The difference between the last two columns of the tables could be regarded as a measure of the error resulting from LECs at  $O(\epsilon^3)$ .

In our present calculation, some polynomials of  $\epsilon$ , such as  $m_\phi^3$ ,  $m_\phi^2\delta$ , appear like nonanalytic in quark mass  $m_q$  at first sight. However, we have checked that the polynomials in our results are analytic in quark mass since the momentum of the external boson at the threshold also contributes a factor  $m_\phi$ , which is simply a kinematical factor. We can extract the nonanalytic contribution with the new functions of  $V$  and  $H_1$  by dropping the analytic terms in the squared brackets in Eqs. (A2, A3).

Comparing the total loop contribution with the sole nonanalytic part in Table IV, we notice that the chiral expansion does become better when appropriate LECs absorb the analytic contribution. There are only 13 channels where the magnitude of the loop correction is smaller than 1/4 in unit of  $m_\phi/f_\phi^2$ . In contrast there are 19 channels where the magnitude of the nonanalytic case is smaller than 1/4 in unit of  $m_\phi/f_\phi^2$ . There are 9 badly convergent channels where the magnitude of the total loop is larger than 2/3. There are only 5 badly convergent channels where the nonanalytic correction is larger than 2/3. We have also checked our previous results for the excited charmed meson and pseudoscalar meson scattering lengths. The nonanalytic terms are smaller than the total loop contributions in the most channels [16].

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TABLE I: The  $\phi$ - $B_3$  threshold  $T$ -matrices order by order in units of fm with  $\alpha' = 0 \pm 1$ .

	$O(\epsilon^1)$	$O(\epsilon^2)$	$O(\epsilon^3)$	Total	Scattering Length	Scattering Length (Nonanalytic Approximation)
$T_{\pi\Lambda_c}^{(1)}$	0	0.9	-0.11	0.79	0.06	0.051
$T_{\pi\Xi_c}^{(1/2)}$	3.2	$0.84 \pm 0.069$	0.15	$4.2 \pm 0.069$	$0.32 \pm 0.0052$	$0.31 \pm 0.0052$
$T_{\pi\Xi_c}^{(3/2)}$	-1.6	$0.84 \pm 0.069$	-0.64	$-1.4 \pm 0.069$	$-0.11 \pm 0.0052$	$-0.1 \pm 0.0052$
$T_{K\Lambda_c}^{(1/2)}$	-3.8	$7. \pm 0.57$	-3.7	$-0.48 \pm 0.57$	$-0.032 \pm 0.038$	$0.059 \pm 0.038$
$T_{K\Xi_c}^{(0)}$	7.6	$6.5 \pm 1.1$	0.78	$15. \pm 1.1$	$0.99 \pm 0.076$	$0.85 \pm 0.076$
$T_{K\Xi_c}^{(1)}$	0	7.5	$4.1+2.8i$	$12. + 2.8i$	$0.77 + 0.18i$	$0.73 + 0.18i$
$T_{\bar{K}\Lambda_c}^{(1/2)}$	3.8	$7. \pm 0.57$	$1.3+4.2i$	$(12. + 4.2i) \pm 0.57$	$(0.79 + 0.27i) \pm 0.038$	$(0.7 + 0.27i) \pm 0.038$
$T_{\bar{K}\Xi_c}^{(0)}$	3.8	$8.1 \mp 0.57$	14	$26. \mp 0.57$	$1.7 \mp 0.038$	$1.5 \mp 0.038$
$T_{\bar{K}\Xi_c}^{(1)}$	-3.8	$7. \pm 0.57$	-3.6	$-0.42 \pm 0.57$	$-0.028 \pm 0.038$	$0.064 \pm 0.038$
$T_{\eta\Lambda_c}^{(0)}$	0	$4. \pm 0.69$	$1.5+3.i$	$(5.5 + 3.i) \pm 0.69$	$(0.35 + 0.19i) \pm 0.044$	$(0.34 + 0.19i) \pm 0.044$
$T_{\eta\Xi_c}^{(1/2)}$	0	$7.8 \pm 0.17$	$0.55+1.5i$	$(8.3 + 1.5i) \pm 0.17$	$(0.54 + 0.098i) \pm 0.011$	$(0.53 + 0.098i) \pm 0.011$

TABLE II: The  $\phi$ - $B_6$  threshold  $T$ -matrices order by order in units of fm with  $\alpha' = 0 \pm 1$ .

	$O(\epsilon^1)$	$O(\epsilon^2)$	$O(\epsilon^3)$	Total	Scattering Length	Scattering Length (Nonanalytic Approximation)
$T_{\pi\Omega_c}^{(1)}$	0	0.46	-1.3	-0.82	-0.062	-0.062
$T_{\pi\Xi_c'}^{(1/2)}$	3.2	$0.64 \mp 0.21$	-0.78	$3.1 \mp 0.21$	$0.23 \mp 0.016$	$0.22 \mp 0.016$
$T_{\pi\Xi_c'}^{(3/2)}$	-1.6	$0.64 \mp 0.21$	-1.6	$-2.5 \mp 0.21$	$-0.19 \mp 0.016$	$-0.19 \mp 0.016$
$T_{\pi\Sigma_c}^{(0)}$	6.5	$0.85 \mp 1.$	1.4	$8.7 \mp 1.$	$0.65 \mp 0.078$	$0.62 \mp 0.078$
$T_{\pi\Sigma_c}^{(1)}$	3.2	0.82	-0.34	3.7	0.28	0.27
$T_{\pi\Sigma_c}^{(2)}$	-3.2	$0.83 \mp 0.42$	-0.91	$-3.3 \mp 0.42$	$-0.25 \mp 0.031$	$-0.24 \mp 0.031$
$T_{K\Omega_c}^{(1/2)}$	7.6	$7. \mp 3.5$	$5.8+8.3i$	$(20. + 8.3i) \mp 3.5$	$(1.4 + 0.56i) \mp 0.23$	$(1.2 + 0.56i) \mp 0.23$
$T_{K\Xi_c'}^{(0)}$	7.6	$3.9 \mp 1.7$	$9.9+8.3i$	$(21. + 8.3i) \mp 1.7$	$(1.4 + 0.56i) \mp 0.12$	$(1.2 + 0.56i) \mp 0.12$
$T_{K\Xi_c'}^{(1)}$	0	$6.9 \mp 1.7$	$-4.3+5.6i$	$(2.6 + 5.6i) \mp 1.7$	$(0.18 + 0.37i) \mp 0.12$	$(0.19 + 0.37i) \mp 0.12$
$T_{K\Sigma_c}^{(1/2)}$	3.8	$2.2 \pm 1.7$	3.3	$9.3 \pm 1.7$	$0.62 \pm 0.12$	$0.53 \pm 0.12$
$T_{K\Sigma_c}^{(3/2)}$	-7.6	$7. \mp 3.5$	-6	$-6.6 \mp 3.5$	$-0.44 \mp 0.23$	$-0.26 \mp 0.23$
$T_{\bar{K}\Omega_c}^{(1/2)}$	-7.6	$7. \mp 3.5$	-4.2	$-4.9 \mp 3.5$	$-0.33 \mp 0.23$	$-0.14 \mp 0.23$
$T_{\bar{K}\Xi_c'}^{(0)}$	3.8	$8.5 \mp 1.7$	-3.1	$9.2 \mp 1.7$	$0.61 \mp 0.12$	$0.54 \mp 0.12$
$T_{\bar{K}\Xi_c'}^{(1)}$	-3.8	$5.4 \mp 1.7$	$-3.3+5.6i$	$(-1.7 + 5.6i) \mp 1.7$	$(-0.11 + 0.37i) \mp 0.12$	$(-0.021 + 0.37i) \mp 0.12$
$T_{\bar{K}\Sigma_c}^{(1/2)}$	11	$8.6 \mp 5.2$	$10.+1.4i$	$(30. + 1.4i) \mp 5.2$	$(2. + 0.092i) \mp 0.35$	$(1.7 + 0.092i) \mp 0.35$
$T_{\bar{K}\Sigma_c}^{(3/2)}$	0	3.8	$-2.+5.6i$	$1.8 + 5.6i$	$0.12 + 0.37i$	$0.12 + 0.37i$
$T_{\eta\Omega_c}^{(0)}$	0	$11. \mp 2.1$	$-1.1+6.1i$	$(10. + 6.1i) \mp 2.1$	$(0.68 + 0.4i) \mp 0.14$	$(0.69 + 0.4i) \mp 0.14$
$T_{\eta\Xi_c'}^{(1/2)}$	0	$6.9 \mp 3.7$	$1.5+7.6i$	$(8.5 + 7.6i) \mp 3.7$	$(0.55 + 0.49i) \mp 0.24$	$(0.54 + 0.49i) \mp 0.24$
$T_{\eta\Sigma_c}^{(1)}$	0	$2.3 \mp 0.53$	$0.43+3.i$	$(2.7 + 3.i) \mp 0.53$	$(0.18 + 0.2i) \mp 0.034$	$(0.18 + 0.2i) \mp 0.034$

TABLE III: The  $\phi$ - $B_6^*$  threshold  $T$ -matrices order by order in units of fm with  $\alpha' = 0 \pm 1$ .

	$O(\epsilon^1)$	$O(\epsilon^2)$	$O(\epsilon^3)$	Total	Scattering Length	Scattering Length (Nonanalytic Approximation)
$T_{\pi\Omega_c^*}^{(1)}$	0	0.46	-1.3	-0.82	-0.062	-0.062
$T_{\pi\Xi_c^*}^{(1/2)}$	3.2	$0.64 \mp 0.21$	$-0.79-0.035i$	$(3.1 - 0.035i) \mp 0.21$	$(0.23 - 0.0027i) \mp 0.016$	$(0.22 - 0.0027i) \mp 0.016$
$T_{\pi\Xi_c^*}^{(3/2)}$	-1.6	$0.64 \mp 0.21$	$-1.6-0.035i$	$(-2.6 - 0.035i) \mp 0.21$	$(-0.19 - 0.0027i) \mp 0.016$	$(-0.19 - 0.0027i) \mp 0.016$
$T_{\pi\Sigma_c^*}^{(0)}$	6.5	$0.97 \mp 1.$	$1.4+0.42i$	$(8.9 + 0.42i) \mp 1.$	$(0.67 + 0.032i) \mp 0.078$	$(0.64 + 0.032i) \mp 0.078$
$T_{\pi\Sigma_c^*}^{(1)}$	3.2	0.72	$-0.43-0.28i$	$3.5 - 0.28i$	$0.27 - 0.021i$	$0.26 - 0.021i$
$T_{\pi\Sigma_c^*}^{(2)}$	-3.2	$0.82 \mp 0.42$	-0.95	$-3.4 \mp 0.42$	$-0.25 \mp 0.031$	$-0.24 \mp 0.031$
$T_{K\Omega_c^*}^{(1/2)}$	7.6	$6.9 \mp 3.5$	$5.8+8.3i$	$(20. + 8.3i) \mp 3.5$	$(1.4 + 0.56i) \mp 0.23$	$(1.2 + 0.56i) \mp 0.23$
$T_{K\Xi_c^*}^{(0)}$	7.6	$4.2 \mp 1.7$	$10.+8.4i$	$(22. + 8.4i) \mp 1.7$	$(1.5 + 0.56i) \mp 0.12$	$(1.3 + 0.56i) \mp 0.12$
$T_{K\Xi_c^*}^{(1)}$	0	$6.5 \mp 1.7$	$-4.5+5.5i$	$(2. + 5.5i) \mp 1.7$	$(0.13 + 0.37i) \mp 0.12$	$(0.14 + 0.37i) \mp 0.12$
$T_{K\Sigma_c^*}^{(1/2)}$	3.8	$2.3 \pm 1.7$	3.4	$9.5 \pm 1.7$	$0.63 \pm 0.12$	$0.54 \pm 0.12$
$T_{K\Sigma_c^*}^{(3/2)}$	-7.6	$6.9 \mp 3.5$	-6.1	$-6.8 \mp 3.5$	$-0.45 \mp 0.23$	$-0.27 \mp 0.23$
$T_{\bar{K}\Omega_c^*}^{(1/2)}$	-7.6	$6.9 \mp 3.5$	-4.2	$-5. \mp 3.5$	$-0.34 \mp 0.23$	$-0.15 \mp 0.23$
$T_{\bar{K}\Xi_c^*}^{(0)}$	3.8	$7.6 \mp 1.7$	$-3.6-0.048i$	$(7.8 - 0.048i) \mp 1.7$	$(0.52 - 0.0032i) \mp 0.12$	$(0.45 - 0.0032i) \mp 0.12$
$T_{\bar{K}\Xi_c^*}^{(1)}$	-3.8	$5.3 \mp 1.7$	$-3.3+5.6i$	$(-1.7 + 5.6i) \mp 1.7$	$(-0.12 + 0.37i) \mp 0.12$	$(-0.022 + 0.37i) \mp 0.12$
$T_{\bar{K}\Sigma_c^*}^{(1/2)}$	11	$8.4 \mp 5.2$	$10.+1.4i$	$(30. + 1.4i) \mp 5.2$	$(2. + 0.092i) \mp 0.35$	$(1.7 + 0.092i) \mp 0.35$
$T_{\bar{K}\Sigma_c^*}^{(3/2)}$	0	3.8	$-1.9+5.6i$	$1.9 + 5.6i$	$0.13 + 0.37i$	$0.12 + 0.37i$
$T_{\eta\Omega_c^*}^{(0)}$	0	$11. \mp 2.1$	$-1.2+6.1i$	$(9.7 + 6.1i) \mp 2.1$	$(0.64 + 0.4i) \mp 0.14$	$(0.64 + 0.4i) \mp 0.14$
$T_{\eta\Xi_c^*}^{(1/2)}$	0	$7.1 \mp 3.7$	$1.6+7.6i$	$(8.7 + 7.6i) \mp 3.7$	$(0.57 + 0.5i) \mp 0.24$	$(0.56 + 0.5i) \mp 0.24$
$T_{\eta\Sigma_c^*}^{(1)}$	0	$2.2 \mp 0.53$	$0.36+3.i$	$(2.5 + 3.i) \mp 0.53$	$(0.16 + 0.2i) \mp 0.034$	$(0.17 + 0.2i) \mp 0.034$

TABLE IV: The  $\phi$ - $B$  threshold  $T$ -matrices at  $O(\epsilon^3)$  in natural units of  $m_\phi/f_\phi^2$ .

	Tree	Loop I	Loop II: triplet	Loop II: sextet	Loop II: excited sextet	Loop total	Nonanalytic part
$T_{\pi\Lambda_c}^{(1)}$	0	-0.15	0	0.04	0.081	-0.033	-0.069
$T_{\pi\Xi}^{(1/2)}$	0.00022	0.085	0	-0.015	-0.025	0.045	0.0018
$T_{\pi\Xi}^{(3/2)}$	-0.0000068	-0.16	0	-0.015	-0.025	-0.2	-0.18
$T_{K\Lambda_c}^{(1/2)}$	-0.0007	-0.48	0	0	0	-0.48	-0.3
$T_{K\Xi}^{(0)}$	0.0024	0.55	0	-0.15	-0.3	0.1	-0.18
$T_{K\Xi}^{(1)}$	0	-0.072+0.36 <i>i</i>	0	0.21	0.4	0.54+0.36 <i>i</i>	0.45+0.36 <i>i</i>
$T_{\bar{K}\Lambda_c}^{(1/2)}$	0.0012	0.17+0.55 <i>i</i>	0	0	0	0.17+0.55 <i>i</i>	-0.013+0.55 <i>i</i>
$T_{\bar{K}\Xi}^{(0)}$	0.0007	0.7	0	0.39	0.75	1.8	1.5
$T_{\bar{K}\Xi}^{(1)}$	-0.0007	-0.56	0	0.03	0.051	-0.47	-0.29
$T_{\eta\Lambda_c}^{(0)}$	0.00037	0.5 <i>i</i>	0	0.085	0.16	0.25+0.5 <i>i</i>	0.21+0.5 <i>i</i>
$T_{\eta\Xi}^{(1/2)}$	0.000092	0.25 <i>i</i>	0	0.03	0.06	0.091+0.25 <i>i</i>	0.072+0.25 <i>i</i>
$T_{\pi\Omega_c}^{(1)}$	0	-0.31	0	-0.058	-0.029	-0.4	-0.39
$T_{\pi\Xi'}^{(1/2)}$	0.0006	-0.22	-0.028	0.0045	0.0038	-0.24	-0.29
$T_{\pi\Xi}^{(3/2)}$	0.000075	-0.47	-0.028	0.0045	0.0038	-0.49	-0.46
$T_{\pi\Sigma_c}^{(0)}$	0.00052	0.17	0.058	0.12	0.083	0.43	0.3
$T_{\pi\Sigma_c}^{(1)}$	0.0013	0.0084	-0.039	-0.047	-0.03	-0.11	-0.13
$T_{\pi\Sigma_c}^{(2)}$	0.00015	-0.32	0	0.018	0.015	-0.28	-0.24
$T_{K\Omega_c}^{(1/2)}$	0.0062	0.34+1.1 <i>i</i>	0	0.28	0.14	0.75+1.1 <i>i</i>	0.38+1.1 <i>i</i>
$T_{K\Xi'}^{(0)}$	0.0036	1.2+1.1 <i>i</i>	-0.079	0.13	0.067	1.3+1.1 <i>i</i>	0.91+1.1 <i>i</i>
$T_{K\Xi'}^{(1)}$	0.0026	-0.7+0.73 <i>i</i>	0.17	-0.019	-0.011	-0.56+0.73 <i>i</i>	-0.54+0.73 <i>i</i>
$T_{K\Sigma_c}^{(1/2)}$	0.0013	0.77	0	-0.22	-0.12	0.43	0.26
$T_{K\Sigma_c}^{(3/2)}$	-0.0026	-1.1	0	0.21	0.11	-0.78	-0.43
$T_{\bar{K}\Omega_c}^{(1/2)}$	-0.0026	-0.97	0	0.28	0.14	-0.55	-0.19
$T_{\bar{K}\Xi'}^{(0)}$	0.0065	-0.56	0.29	-0.091	-0.05	-0.41	-0.55
$T_{\bar{K}\Xi'}^{(1)}$	-0.0013	-0.56+0.73 <i>i</i>	0.045	0.053	0.028	-0.43+0.73 <i>i</i>	-0.25+0.73 <i>i</i>
$T_{\bar{K}\Sigma_c}^{(1/2)}$	0.0092	0.79+0.18 <i>i</i>	0	0.36	0.19	1.3+0.18 <i>i</i>	0.78+0.18 <i>i</i>
$T_{\bar{K}\Sigma_c}^{(3/2)}$	0	-0.14+0.73 <i>i</i>	0	-0.076	-0.042	-0.26+0.73 <i>i</i>	-0.26+0.73 <i>i</i>
$T_{\eta\Omega_c}^{(0)}$	0.0039	1 <i>i</i>	0.14	-0.21	-0.11	-0.18+1 <i>i</i>	-0.16+1 <i>i</i>
$T_{\eta\Xi'}^{(1/2)}$	0.0011	1.3 <i>i</i>	-0.13	0.25	0.13	0.25+1.3 <i>i</i>	0.23+1.3 <i>i</i>
$T_{\eta\Sigma_c}^{(1)}$	0.00098	0.5 <i>i</i>	0.07	-0.00035	0.00011	0.07+0.5 <i>i</i>	0.079+0.5 <i>i</i>
$T_{\pi\Omega_c^*}^{(1)}$	0	-0.31	0	-0.014	-0.073	-0.4	-0.4
$T_{\pi\Xi^*}^{(1/2)}$	0.0011	-0.22	-0.03-0.011 <i>i</i>	-0.00017	0.0056	-0.25-0.011 <i>i</i>	-0.29-0.011 <i>i</i>
$T_{\pi\Xi^*}^{(3/2)}$	0.0004	-0.47	-0.03-0.011 <i>i</i>	-0.00017	0.0056	-0.49-0.011 <i>i</i>	-0.47-0.011 <i>i</i>
$T_{\pi\Sigma_c^*}^{(0)}$	-0.0016	0.17	0.12+0.13 <i>i</i>	0.008	0.14	0.44+0.13 <i>i</i>	0.32+0.13 <i>i</i>
$T_{\pi\Sigma_c^*}^{(1)}$	0.0042	0.0084	-0.081-0.087 <i>i</i>	-0.0064	-0.059	-0.14-0.087 <i>i</i>	-0.16-0.087 <i>i</i>
$T_{\pi\Sigma_c^*}^{(2)}$	0.0008	-0.32	0	-0.00068	0.022	-0.29	-0.25
$T_{K\Omega_c^*}^{(1/2)}$	0.01	0.34+1.1 <i>i</i>	0	0.066	0.34	0.75+1.1 <i>i</i>	0.39+1.1 <i>i</i>
$T_{K\Xi^*}^{(0)}$	0.0012	1.2+1.1 <i>i</i>	-0.03+0.0063 <i>i</i>	0.028	0.16	1.3+1.1 <i>i</i>	0.95+1.1 <i>i</i>
$T_{K\Xi^*}^{(1)}$	0.0089	-0.7+0.73 <i>i</i>	0.13-0.0021 <i>i</i>	-0.0039	-0.024	-0.6+0.73 <i>i</i>	-0.57+0.73 <i>i</i>
$T_{K\Sigma_c^*}^{(1/2)}$	0.0006	0.77	0	-0.048	-0.28	0.45	0.26
$T_{K\Sigma_c^*}^{(3/2)}$	-0.0012	-1.1	0	0.049	0.27	-0.79	-0.43
$T_{\bar{K}\Omega_c^*}^{(1/2)}$	-0.0012	-0.97	0	0.066	0.34	-0.56	-0.19
$T_{\bar{K}\Xi^*}^{(0)}$	0.018	-0.56	0.21-0.0063 <i>i</i>	-0.02	-0.11	-0.49-0.0063 <i>i</i>	-0.61-0.0063 <i>i</i>
$T_{\bar{K}\Xi^*}^{(1)}$	-0.0006	-0.56+0.73 <i>i</i>	0.051+0.0021 <i>i</i>	0.012	0.067	-0.43+0.73 <i>i</i>	-0.24+0.73 <i>i</i>
$T_{\bar{K}\Sigma_c^*}^{(1/2)}$	0.015	0.79+0.18 <i>i</i>	0	0.081	0.45	1.3+0.18 <i>i</i>	0.78+0.18 <i>i</i>
$T_{\bar{K}\Sigma_c^*}^{(3/2)}$	0	-0.14+0.73 <i>i</i>	0	-0.016	-0.095	-0.25+0.73 <i>i</i>	-0.26+0.73 <i>i</i>
$T_{\eta\Omega_c^*}^{(0)}$	0.014	1 <i>i</i>	0.11	-0.051	-0.26	-0.2+1 <i>i</i>	-0.18+1 <i>i</i>
$T_{\eta\Xi^*}^{(1/2)}$	0.00084	1.3 <i>i</i>	-0.11+0.0013 <i>i</i>	0.057	0.31	0.26+1.3 <i>i</i>	0.24+1.3 <i>i</i>
$T_{\eta\Sigma_c^*}^{(1)}$	0.0034	0.5 <i>i</i>	0.057+0.0017 <i>i</i>	-0.00021	-0.00043	0.057+0.5 <i>i</i>	0.069+0.5 <i>i</i>